§ 2.2 Separable Equations

Definition 2.2.1: Separable Equations

A first-order differential equation of the form

$$\frac{dy}{dx} = g(x)h(y)$$

is said to be **separable** or to have **separable variables**.

Example: Determine whether the following equations are separable or nonseparable.

a) $\frac{dy}{dx} = (\sin x)y^3 e^{5x+2y}$

b) $\frac{dy}{dx} = e^{7x} + y^2$

To solve separable equations, first rewrite the equation as $p(y)\frac{dy}{dx} = g(x)$ (where $p(y) = \frac{1}{h(y)}$) which gives us $p(y)dy = h(x)dx$. Then integrate both sides $\int p(y)dy = \int h(x)dx$.

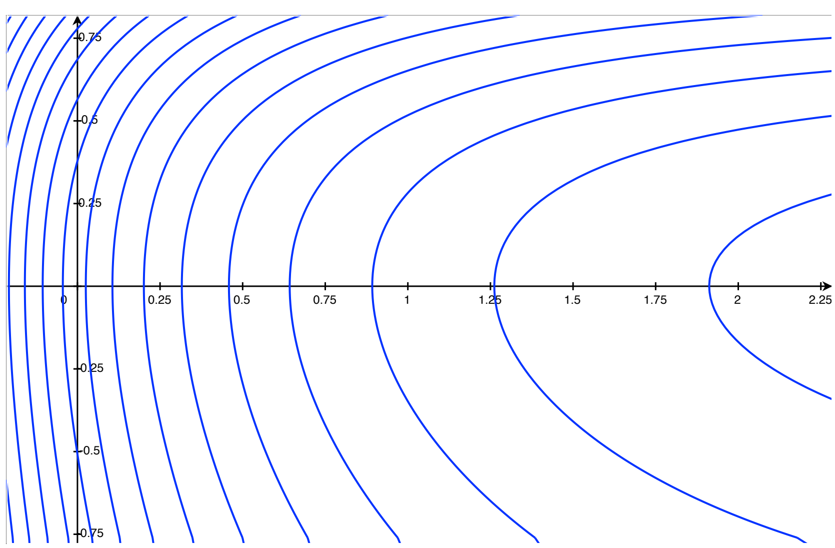
Note:

1. After doing the integrations you will have an implicit solution that you can hopefully solve for the explicit solution, $y(x)$. Note that it won't always be possible to solve for an explicit solution.
2. There is no need to use two constants in the integration.

Example: Solve. $\frac{dy}{dx} = 6y^2 x$

Example: Solve. $e^x y\dfrac{dy}{dx} = e^{-y} + e^{-2x-y}$

Recall from multivariable calculus that for a function of two variables $z = G(x, y)$ the *two dimensional curves* defined by $G(x, y) = c$, where c is a constant, are called the *level curves* of the function. Some of the level curves of the function $G(x, y) = ye^y - e^y + e^{-x} + \frac{1}{3}e^{-3x}$ are shown below.



Example: Solve. $\dfrac{dx}{dt} = x^2 - 9$

Note: $\dfrac{dx}{dt} = x^2 - 9 = (x+3)(x-3)$ has two constant (equilibrium) solutions $x =$ __________ and $x =$________ . The solution $x =$ __________ is a member of the family of solutions corresponding to $c = 0$. However, $x =$ __________ is a singular solution (it cannot be obtained by any choice of c).

Example: Find an explicit solution of the IVP. Determine the interval of definition, *I*.

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}; \qquad y(1) = -2$$