

Quiz 2

Math 2C – Differential Equations

Due Date: Wednesday 15 March

Name: _____

Show all work for full credit. Do work on separate sheets of paper.

1. A tank with a capacity of 500 gallons contains 200 gallons of water with 100 lb of salt dissolved in it. Brine containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min.
 - a) When will the tank overflow?
 - b) Find the amount of salt in the tank at the moment it overflows.
2. A dead body was found within a closed room of a house where the temperature was a constant 70°F . At the time of discovery the core temperature of the body was determined to be 85°F . One hour later a second measurement showed that the core temperature of the body was 80°F . Assume that the core temperature of the body at the time of death was 98.6°F . Determine how many hours elapsed before the body was found.
3. Consider the equation $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$.
 - a) Your friend correctly checks that both $L(e^{3x}) = 0$ and $L(e^{-x}) = 0$. He claims that the set of functions $\{e^{3x}, e^{-x}\}$ is the fundamental set of solutions for the differential equation. Briefly explain why he cannot be correct.
 - b) Verify that $y = e^{2x}$ is a solution to the differential equation.
 - c) State the correct fundamental set and show that your set of functions is linearly independent.
 - d) Write the general solution of the differential equation.
 - e) Show that the function $y = 2x + \frac{4}{3}$ is a solution of $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 12x + 10$.
 - f) Write the general solution of the equation $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 12x + 10$.