Quiz 1 Name:_______________________________
Math 2C – Differential Equations
Due Date: Wednesday 22 February

Show all work for full credit. Do work on separate sheets of paper.

You will need to download the program "Slopefield" onto your TI-84 calculator. The Slopefield program can be transferred from the Math Lab's computer to yours or from the Math Lab calculators to your calculator. See Deanna Souza or myself for help with this. You will also need to go to the website http://math.rice.edu/~dfield/dfpp.html, in order to get the direction fields for different differential equations. However, this program does not graph your solution to see if it works. Rather, if you click on or enter an initial condition, it will plot the solution using the direction field.

1. Consider the differential equation:

$$y' - \frac{1}{2}y = 2\cos t \;\; ; \;\; y(0) = a$$

a) Use the web address printed above. Click to download and save dfield.jar. After downloading, find the directory where you saved dfield.jar and double-click on the downloaded file. Enter y' into the dfield equation window (note that in their template, *x* is a function of *t*). Start with a window of *t* from -4 to 10, and *y* from -10 to 10. In the upper part of the graph, click on "Solution", and then "Keyboard Input of an Initial Value." Let *t* be zero and pick a few different values of *a*. The software will plot the solution.
   - How do solutions appear to behave as *t* becomes large? How does the end behavior of *y* depend on the initial condition (the value of *a*)? Print out the direction field and include it in your work for part a).

b) Let a_0 be the value of *a* for which the transition from one type of end behavior to another occurs. (In other words, where it changes form $y \to \infty$ to $y \to -\infty$ as $t \to \infty$) Estimate the value of a_0 from the direction field and indicate that point on your printout from part a).

c) Now solve the IVP and find the value of a_0 exactly, the point at which the end behavior changes. Let the initial condition be a_0 and plot the solution on the direction field. Pick values for *a* that are larger and smaller than a_0, and plot the solutions on the same direction field. Print out the graph of these curves on the direction field.

d) Go to Slopefield in your calculator and enter the differential equation. Let *x* range from -1 to 5 with a scale of 0.5, and let *y* range from -5 to 5 with a scale of 1. View the direction field on your calculator. Hit enter to get back to the menu, then go to 5: Graph Original Equation. Enter your solution from step c) for *y*, and verify that it follows the flow of the direction field. If your solution doesn't follow the flow, either you entered it incorrectly in your calculator or you didn't get the correct solution.

2. Consider the differential equation: $\left(1+t^2\right)y' + 4ty = \left(1+t^2\right)^{-2}$

a) Create a direction field for the DE and print it out.
b) Based on the direction field, describe how solutions behave as $t \to \infty$.
c) Solve the DE and use your result to determine how solutions behave as $t \to \infty$.

3. Consider the IVP: $\dfrac{dr}{dt} = \dfrac{r^2}{t}$, $r(1) = 2$

   Find an explicit solution and determine the exact interval of definition, *I*.