Math 2C

Final Review

The final exam will be on **Wednesday, May 24th, 10:30 am-12:30 pm**. Below is a list of topics that may appear on the final. Go rework (or partially rework) problems to reacquaint yourself with these topics. You should also thoroughly understand all questions from the exams and quizzes. The end of this sheet has more review problems from selected chapters. You may use one side of one 8.5" by 11" page of notes (no specific examples). Prepare well and good luck!

1. Solve an initial value problem (IVP) for any of the types of ODEs that follow.

2. **2.1 – Slope Fields:** Provided the slope field of a first order DE, sketch solutions to an IVP.

3. **2.2 – Separable Equations:** Recognize a first-order ODE as separable, and use the technique from section 2.2 to find general solutions to the ODE. Recall, these ODEs may not be linear and may have variable coefficients.

4. **2.3 – Linear Equations:** Recognize a first-order ODE as linear, and solve the ODE using an integrating factor $\mu(x) = e^{\int P(x)dx}$.

5. **2.4 – Exact Equations:** Recognize a first-order ODE as potentially exact, then test for exactness. If exact, find the function $f(x,y)$ whose differential produces the left hand side of the exact equation in definition 2.4.1. What is the solution of the exact equation?

6. **3.1 – Linear Models:** Solve application problems presented in section 3.1 (population growth, radioactive decay, proportional growth and decay, Newton's Law of cooling/warming, tank mixtures).

7. **4.1 – Preliminary Theory:** Reacquaint yourself with Existence and Uniqueness as it pertains to linear n^{th}-order differential equations, superposition principles, linear dependence/independence, fundamental sets of solutions, y_c, y_p, general solutions (to homogeneous and nonhomogeneous DEs), and the Wronskian.

   a) Determine if a set of functions is linearly independent on an interval I.
   b) Verify that a set of functions form a fundamental set of solutions to a given linear n^{th}-order homogeneous ODE. Use the set to form the general solution.
   c) Verify that a given two-parameter family of functions is the general solution of a given linear 2^{nd}-order nonhomogeneous ODE.

8. **4.3 – Homogeneous Linear DEs with Constant Coefficients:** Using the auxiliary equation, solve homogeneous linear DEs with constant coefficients. The focus of this section will be DEs of order 3 or higher.

9. **4.4 – Undetermined Coefficients:** Use the method of undetermined coefficients to solve nonhomogeneous linear DEs with constant coefficients. You may be asked to:
   a) Give only the simplest form of the particular solution y_p (thinking of "repeats").
   b) Give the form of the particular solution, then solve the DE.
   c) Solve an IVP using the above method.

10. **4.6 – Variation of Parameters:** Use variation of parameters to solve nonhomogeneous linear DEs with constant coefficients. It may be possible to solve the DE using undetermined coefficients, but I may specifically ask you to solve it using variation of parameters.

11. **5.1 – Spring/Mass Systems:** Find an equation to describe the position of a mass in a spring/mass system using techniques derived in lecture. Be able to give your solution in the form $x(t) = A\sin(\omega t + \phi)$.

    a) Three cases: free undamped, free damped, driven motion
    b) Determine (quickly) if a damped system is underdamped, critically damped, or overdamped.
    c) Use your equation of motion (for the undamped case) to answer questions about the mass.

12. **6.2 – Solutions About Ordinary Points:** Find power series solutions to linear second-order DEs with variable coefficients.

13. **7.1 – Definition of the Laplace Transform:** Apply the definition of the Laplace transform to evaluate $\mathrm{L}\{f(t)\}$ for a given function $f(t)$.

14. **7.2 – Inverse Laplace Transforms and Transforms of Derivatives:**

    a) Find the inverse of a Laplace transform that has the form of those transforms found in the table under Theorem 7.2.1.
    b) Know the Laplace transform of y' and y''.
    c) Use Laplace transforms to solve DEs.

15. **7.3 – Translation on the *s*-axis**
    a) Use the First Translation Theorem to find the Laplace transform of a "shifted" function.
    b) Find the inverse of a Laplace transform of a "shifted" function.
    c) Use Laplace transforms to solve DEs.

16. **8.2 – Homogeneous Linear Systems:** Find the general solution to homogeneous systems with distinct real eigenvalues and complex eigenvalues.

Some review problems (in addition to understanding all assigned HW and class examples) are found in the review sets at the end of the chapters.

Chapter 2: Page 81 / 19, 22, 23, 25 (linear), 26, 28
Chapter 3: See assigned HW
Chapter 4: Page 190 / 2, 5, 8, 9 10, 13a, 14, 15 – 20, 22 – 26, 35 – 38
Chapter 5: Page 228 / 1, 2, 3, 12, 19
Chapter 6: Page 271 / 10, 11
Chapter 7: Page 321 / 7 – 10, 13-15, 17, 21, 33
Page 289 / 19, 26, 33, 38
Page 297 / 15, 27
Chapter 8: See assigned HW for 8.2 (skip the problems with repeated real eigenvalues)