means of populations. In Chapter 7 we will apply it when we use sample data to test claims made about population means. Such applications of estimating population parameters and testing claims are extremely important uses of statistics, and the central limit theorem makes them possible.

5-5 Basic Skills and Concepts

Using the Central Limit Theorem. In Exercises 1–6, assume that men’s weights are normally distributed with a mean given by \( \mu = 172 \) lb and a standard deviation given by \( \sigma = 29 \) lb (based on data from the National Health Survey).

1. a. If 1 man is randomly selected, find the probability that his weight is less than 167 lb.
   b. If 36 men are randomly selected, find the probability that they have a mean weight less than 167 lb.

2. a. If 1 man is randomly selected, find the probability that his weight is greater than 180 lb.
   b. If 100 men are randomly selected, find the probability that they have a mean weight greater than 180 lb.

3. a. If 1 man is randomly selected, find the probability that his weight is between 170 lb and 175 lb.
   b. If 64 men are randomly selected, find the probability that they have a mean weight between 170 and 175 lb.

4. a. If 1 man is randomly selected, find the probability that his weight is between 100 lb and 165 lb.
   b. If 81 men are randomly selected, find the probability that they have a mean weight between 100 lb and 165 lb.

5. a. If 25 men are randomly selected, find the probability that they have a mean weight greater than 160 lb.
   b. Why can the central limit theorem be used in part (a), even though the sample size does not exceed 30?

6. a. If 4 men are randomly selected, find the probability that they have a mean weight between 160 lb and 180 lb.
   b. Why can the central limit theorem be used in part (a), even though the sample size does not exceed 30?

7. Redesign of Ejection Seats In the Chapter Problem, it was noted that engineers were redesigning fighter jet ejection seats to better accommodate women. Before women became fighter jet pilots, the ACES-II ejection seats were designed for men weighing between 140 lb and 211 lb. The population of women has normally distributed weights with a mean of 143 lb and a standard deviation of 29 lb (based on data from the National Health Survey).
   a. If 1 woman is randomly selected, find the probability that her weight is between 140 lb and 211 lb.
   b. If 36 different women are randomly selected, find the probability that their mean weight is between 140 lb and 211 lb.
   c. When redesigning the fighter jet ejection seats to better accommodate women, which probability is more relevant: the result from part (a) or the result from part (b)? Why?
8. **Designing Motorcycle Helmets** Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 in. and a standard deviation of 1.0 in. (based on anthropometric survey data from Gordon, Churchill, et al.).

   a. If one male is randomly selected, find the probability that his head breadth is less than 6.2 in.
   b. The Safeguard Helmet company plans an initial production run of 100 helmets. Find the probability that 100 randomly selected men have a mean head breadth less than 6.2 in.
   c. The production manager sees the result from part (b) and reasons that all helmets should be made for men with head breadths less than 6.2 in., because they would fit all but a few men. What is wrong with that reasoning?

9. **Designing a Roller Coaster** The Rock 'n' Roller Coaster at Disney–MGM Studios in Orlando has two seats in each row. When designing that roller coaster, the total width of the two seats in each row had to be determined. In the “worst case” scenario, both seats are occupied by men. Men have hip breadths that are normally distributed with a mean of 14.4 in. and a standard deviation of 1.0 in. (based on anthropometric survey data from Gordon, Churchill, et al). Assume that two male riders are randomly selected.

   a. Find the probability that their mean hip width is greater than 16.0 in.
   b. If each row of two seats is designed to fit two men only if they have a mean hip breadth of 16.0 in. or less, would too many riders be unable to fit? Does this design appear to be acceptable?

10. **Uniform Random-Number Generator** The random-number generator on the TI-83 Plus calculator and many other calculators and computers yields numbers from a uniform distribution of values between 0 and 1, with a mean of 0.500 and a standard deviation of 0.289. If 100 random numbers are generated, find the probability that their mean is greater than 0.57. Would it be unusual to generate 100 such numbers and get a mean greater than 0.57? Why or why not?

11. **Amounts of Coke** Assume that cans of Coke are filled so that the actual amounts have a mean of 12.00 oz and a standard deviation of 0.11 oz.

   a. Find the probability that a sample of 36 cans will have a mean amount of at least 12.19 oz, as in Data Set 17 in Appendix B.
   b. Based on the result from part (a), is it reasonable to believe that the cans are actually filled with a mean of 12.00 oz? If the mean is not 12.00 oz, are consumers being cheated?

12. **IQ Scores** Membership in Mensa requires an IQ score above 131.5. Nine candidates take IQ tests, and their summary results indicated that their mean IQ score is 133. (IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.)

   a. If 1 person is randomly selected, find the probability of getting someone with an IQ score of at least 133.
   b. If 9 people are randomly selected, find the probability that their mean IQ score is at least 133.
   c. Although the summary results are available, the individual IQ test scores have been lost. Can it be concluded that all 9 candidates have IQ scores above 133 so that they are all eligible for Mensa membership?

13. **Mean Replacement Times** The manager of the Portland Electronics store is concerned that his suppliers have been giving him TV sets with lower than average quality. His research shows that replacement times for TV sets have a mean of 8.2 years
and a standard deviation of 1.1 years (based on data from “Getting Things Fixed,” Consumer Reports). He then randomly selects 50 TV sets sold in the past and finds that the mean replacement time is 7.8 years.

a. Assuming that TV replacement times have a mean of 8.2 years and a standard deviation of 1.1 years, find the probability that 50 randomly selected TV sets will have a mean replacement time of 7.8 years or less.

b. Based on the result from part (a), does it appear that the Portland Electronics store has been given TV sets with lower than average quality?

14. Blood Pressure For women aged 18–24, systolic blood pressures (in mm Hg) are normally distributed with a mean of 114.8 and a standard deviation of 13.1 (based on data from the National Health Survey). Hypertension is commonly defined as a systolic blood pressure above 140.

a. If a woman between the ages of 18 and 24 is randomly selected, find the probability that her systolic blood pressure is greater than 140.

b. If 4 women in that age bracket are randomly selected, find the probability that their mean systolic blood pressure is greater than 140.

c. Given that part (b) involves a sample size that is not larger than 30, why can the central limit theorem be used?

d. If a physician is given a report stating that 4 women have a mean systolic blood pressure below 140, can she conclude that none of the women have hypertension (with a blood pressure greater than 140)?

15. Reduced Nicotine in Cigarettes The amounts of nicotine in Dytusoon cigarettes have a mean of 0.941 g and a standard deviation of 0.313 g (based on Data Set 5 in Appendix B). The Huntington Tobacco Company, which produces Dytusoon cigarettes, claims that it has now reduced the amount of nicotine. The supporting evidence consists of a sample of 40 cigarettes with a mean nicotine amount of 0.882 g.

a. Assuming that the given mean and standard deviation have not changed, find the probability of randomly selecting 40 cigarettes with a mean of 0.882 g or less.

b. Based on the result from part (a), is it valid to claim that the amount of nicotine is lower? Why or why not?

16. Coaching for the SAT Test Scores for men on the verbal portion of the SAT-I test are normally distributed with a mean of 509 and a standard deviation of 112 (based on data from the College Board). Randomly selected men are given the Columbia Review Course before taking the SAT test. Assume that the course has no effect.

a. If 1 of the men is randomly selected, find the probability that his score is at least 590.

b. If 16 of the men are randomly selected, find the probability that their mean score is at least 590.

c. In finding the probability for part (b), why can the central limit theorem be used even though the sample size does not exceed 30?

d. If the random sample of 16 men does result in a mean score of 590, is there strong evidence to support the claim that the course is actually effective? Why or why not?

17. Overloading of Waste Disposal Facility The town of Newport operates a rubbish waste disposal facility that is overloaded if its 4872 households discard waste with weights having a mean that exceeds 27.88 lb in a week. For many different weeks, it is found that the samples of 4872 households have weights that are normally distributed with a mean of 27.44 lb and a standard deviation of 12.46 lb (based on data
from the Garbage Project at the University of Arizona). What is the proportion of weeks in which the waste disposal facility is overloaded? Is this an acceptable level, or should action be taken to correct a problem of an overloaded system?

18. **Labeling of M&M Packages** M&M plain candies have a mean weight of 0.9147 g and a standard deviation of 0.0369 g (based on Data Set 19 in Appendix B). The M&M candies used in Data Set 19 came from a package containing 1498 candies, and the package label stated that the net weight is 1361 g. (If every package has 1498 candies, the mean weight of the candies must exceed 1361/1498 = 0.9085 g for the net contents to weigh at least 1361 g.)

   a. If 1 M&M plain candy is randomly selected, find the probability that it weighs more than 0.9085 g.
   b. If 1498 M&M plain candies are randomly selected, find the probability that their mean weight is at least 0.9085 g.
   c. Given these results, does it seem that the Mars Company is providing M&M consumers with the amount claimed on the label?

19. **Elevator Design** Women’s weights are normally distributed with a mean of 143 lb and a standard deviation of 29 lb, and men’s weights are normally distributed with a mean of 172 lb and a standard deviation of 29 lb (based on data from the National Health Survey). You need to design an elevator for the Westport Shopping Center, and it must safely carry 16 people. Assuming a worst case scenario of 16 male passengers, find the maximum total allowable weight if we want a 0.975 probability that this maximum will not be exceeded when 16 males are randomly selected.

20. **Seating Design** You need to build a bench that will seat 18 male college football players, and you must first determine the length of the bench. Men have hip breadths that are normally distributed with a mean of 14.4 in. and a standard deviation of 1.0 in.

   a. What is the minimum length of the bench if you want a 0.975 probability that it will fit the combined hip breadths of 18 randomly selected men?
   b. What would be wrong with actually using the result from part (a) as the bench length?

### 5-5 Beyond the Basics

21. **Correcting for a Finite Population** The Boston Women’s Club needs an elevator limited to 8 passengers. The club has 120 women members with weights that approximate a normal distribution with a mean of 143 lb and a standard deviation of 29 lb. (*Hint:* See the discussion of the finite population correction factor.)

   a. If 8 different members are randomly selected, find the probability that their total weight will not exceed the maximum capacity of 1300 lb.
   b. If we want a 0.99 probability that the elevator will not be overloaded whenever 8 members are randomly selected as passengers, what should be the maximum allowable weight?

22. **Population Parameters** A population consists of these values: 2, 3, 6, 8, 11, 18.

   a. Find $\mu$ and $\sigma$.
   b. List all samples of size $n = 2$ that can be obtained without replacement.
   c. Find the population of all values of $\bar{x}$ by finding the mean of each sample from part (b).

   *continued*
d. Find the mean $\mu_\tau$ and standard deviation $\sigma_\tau$ for the population of sample means found in part (c).

e. Verify that

$$\mu_\tau = \mu \quad \text{and} \quad \sigma_\tau = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N - n}{N - 1}}$$

23. **Uniform Random-Number Generator** In Exercise 10 it was noted that many calculators and computers have a random-number generator that yields numbers from a uniform distribution of values between 0 and 1, with a mean of 0.500 and a standard deviation of 0.289. If 100 random numbers are generated, find the probability that their mean is between 0.499 and 0.501. If we did generate 100 such numbers and found the mean to be between 0.499 and 0.501, can we conclude that the result is “unusual” so that the random-number generator is somehow defective? Why or why not?