4-3 Basic Skills and Concepts

Identifying Binomial Distributions. In Exercises 1–8, determine whether the given procedure results in a binomial distribution. For those that are not binomial, identify at least one requirement that is not satisfied.

1. Surveying people by asking them what they think of the current president
2. Surveying 1012 people and recording whether there is a “should not” response to the question: “Do you think the cloning of humans should or should not be allowed?”
3. Rolling a fair die 50 times
4. Rolling a loaded die 50 times and finding the number of times that 5 occurs
5. Recording the genders of 250 newborn babies
6. Determining whether each of 3000 heart pacemakers is acceptable or defective
7. Spinning a roulette wheel 12 times
8. Spinning a roulette wheel 12 times and finding the number of times that the outcome is an odd number

Finding Probabilities When Guessing Answers Multiple-choice questions each have five possible answers, one of which is correct. Assume that you guess the answers to three such questions.
10. **Finding Probabilities When Guessing Answers** A test consists of multiple-choice questions, each having four possible answers, one of which is correct. Assume that you guess the answers to six such questions.

   a. Use the multiplication rule to find the probability that the first two guesses are wrong and the last four guesses are correct. That is, find $P(WWCCCC)$, where $C$ denotes a correct answer and $W$ denotes a wrong answer.

   b. Beginning with $WWCCCC$, make a complete list of the different possible arrangements of two wrong answers and four correct answers, then find the probability for each entry in the list.

   c. Based on the preceding results, what is the probability of getting exactly four correct answers when six guesses are made?

Using Table A-1. In Exercises 11–16, assume that a procedure yields a binomial distribution with a trial repeated $n$ times. Use Table A-1 to find the probability of $x$ successes given the probability $p$ of success on a given trial.

**Using the Binomial Probability Formula.** In Exercises 17–20, assume that a procedure yields a binomial distribution with a trial repeated $n$ times. Use the binomial probability formula to find the probability of $x$ successes given the probability $p$ of success on a single trial.

Using Computer Results. In Exercises 21–24, refer to the Minitab display in the margin. The probabilities were obtained by entering the values of $n = 6$ and $p = 0.723$. There is a 0.723 probability that a randomly selected American Airlines flight will arrive on time (based on data from the Department of Transportation). In each case, assume that six American Airlines flights are randomly selected and find the indicated probability.
24. Find the probability that at least one American Airlines flight arrives on time. Is it unusual to not get at least one of six American Airlines flights arriving on time?

25. **Color Blindness** Nine percent of men and 0.25% of women cannot distinguish between the colors red and green. This is the type of color blindness that causes problems with traffic signals. If six men are randomly selected for a study of traffic signal perceptions, find the probability that exactly two of them cannot distinguish between red and green.

26. **Acceptance Sampling** The Telektronic Company purchases large shipments of fluorescent bulbs and uses this acceptance sampling plan: Randomly select and test 24 bulbs, then accept the whole batch if there is only one or none that doesn’t work. If a particular shipment of thousands of bulbs actually has a 4% rate of defects, what is the probability that this whole shipment will be accepted?

27. **IRS Audits** The Hemingway Financial Company prepares tax returns for individuals. (Motto: “We also write great fiction.”) According to the Internal Revenue Service, individuals making $25,000–$50,000 are audited at a rate of 1%. The Hemingway Company prepares five tax returns for individuals in that tax bracket, and three of them are audited.
   a. Find the probability that when five people making $25,000–$50,000 are randomly selected, exactly three of them are audited.
   b. Find the probability that at least three are audited.
   c. Based on the preceding results, what can you conclude about the Hemingway customers? Are they just unlucky, or are they being targeted for audits?

28. **Directory Assistance** An article in *USA Today* stated that “Internal surveys paid for by directory assistance providers show that even the most accurate companies give out wrong numbers 15% of the time.” Assume that you are testing such a provider by making 10 requests and also assume that the provider gives the wrong telephone number 15% of the time.
   a. Find the probability of getting one wrong number.
   b. Find the probability of getting at most one wrong number.
   c. If you do get at most one wrong number, does it appear that the rate of wrong numbers is not 15%, as claimed?

29. **Overbooking Flights** Air America has a policy of booking as many as 15 persons on an airplane that can seat only 14. (Past studies have revealed that only 85% of the booked passengers actually arrive for the flight.) Find the probability that if Air America books 15 persons, not enough seats will be available. Is this probability low enough so that overbooking is not a real concern for passengers?

30. **Drug Reaction** In a clinical test of the drug Viagra, it was found that 4% of those in a placebo group experienced headaches.
   a. Assuming that the same 4% rate applies to those taking Viagra, find the probability that among eight Viagra users, three experience headaches.
   b. Assuming that the same 4% rate applies to those taking Viagra, find the probability that among eight randomly selected users of Viagra, all eight experienced a headache.
   c. If all eight Viagra users were to experience a headache, would it appear that the headache rate for Viagra users is different than the 4% rate for those in the placebo group? Explain.
31. **TV Viewer Surveys** The CBS television show *60 Minutes* has been successful for many years. That show recently had a share of 20, meaning that among the TV sets in use, 20% were tuned to *60 Minutes* (based on data from Nielsen Media Research). Assume that an advertiser wants to verify that 20% share value by conducting its own survey, and a pilot survey begins with 10 households having TV sets in use at the time of a *60 Minutes* broadcast.
   a. Find the probability that none of the households are tuned to *60 Minutes*.
   b. Find the probability that at least one household is tuned to *60 Minutes*.
   c. Find the probability that at most one household is tuned to *60 Minutes*.
   d. If at most one household is tuned to *60 Minutes*, does it appear that the 20% share value is wrong? Why or why not?

32. **Affirmative Action Programs** A study was conducted to determine whether there were significant differences between medical students admitted through special programs (such as affirmative action) and medical students admitted through the regular admissions criteria. It was found that the graduation rate was 94% for the medical students admitted through special programs (based on data from the *Journal of the American Medical Association*).
   a. If 10 of the students from the special programs are randomly selected, find the probability that at least nine of them graduated.
   b. Would it be unusual to randomly select 10 students from the special programs and get only seven that graduate? Why or why not?

33. **Identifying Gender Discrimination** After being rejected for employment, Kim Kelly learns that the Bellevue Advertising Company has hired only two women among the last 20 new employees. She also learns that the pool of applicants is very large, with an approximately equal number of qualified men and women. Help her address the charge of gender discrimination by finding the probability of getting two or fewer women when 20 people are hired, assuming that there is no discrimination based on gender. Does the resulting probability really support such a charge?

34. **Author’s Slot Machine** The author purchased a slot machine that is configured so that there is a 1/2000 probability of winning the jackpot on any individual trial. Although no one would seriously consider tricking the author, suppose that a guest claims that she played the slot machine five times and hit the jackpot twice.
   a. Find the probability of exactly two jackpots in five trials.
   b. Find the probability of at least two jackpots in five trials.
   c. Does the guest’s claim of two jackpots in five trials seem valid? Explain.

35. **Testing Effectiveness of Gender-Selection Technique** The Chapter Problem describes the probability distribution for the number of girls *x* when 14 newborn babies are randomly selected. Assume that another clinical experiment involves 12 newborn babies. Using the same format as Table 4-1, construct a table for the probability distribution that results from 12 births, then determine whether a gender-selection technique appears to be effective if there are 9 girls and 3 boys.

36. **Taking Courses After Graduation** The Market Research Institute found that among employed college graduates aged 30–55 and out of college for at least 10 years, 57% have taken college courses after graduation (as reported in *USA Today*). If you randomly select five college graduates aged 30–55 and out of college for at least 10 years, and you find that only one of them has taken college courses after graduation, should you believe that the 57% rate is wrong? Explain.
4-3 Beyond the Basics

37. If a procedure meets all the conditions of a binomial distribution except that the number of trials is not fixed, then the \textbf{geometric distribution} can be used. The probability of getting the first success on the \(x\)th trial is given by
\[
P(x) = p(1-p)^{x-1}
\]
where \(p\) is the probability of success on any one trial. Assume that the probability of a defective computer component is 0.2. Find the probability that the first defect is found in the seventh component tested.

38. If we sample from a small finite population without replacement, the binomial distribution should not be used because the events are not independent. If sampling is done without replacement and the outcomes belong to one of two types, we can use the \textbf{hypergeometric distribution}. If a population has \(A\) objects of one type, while the remaining \(B\) objects are of the other type, and if \(n\) objects are sampled without replacement, then the probability of getting \(x\) objects of type \(A\) and \(n-x\) objects of type \(B\) is
\[
P(x) = \frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}
\]
In Lotto 54, a bettor selects six numbers from 1 to 54 (without repetition), and a winning six-number combination is later randomly selected. Find the probability of getting
\begin{enumerate}
  \item All six winning numbers
  \item Exactly five of the winning numbers
  \item Exactly three of the winning numbers
  \item No winning numbers
\end{enumerate}

39. The binomial distribution applies only to cases involving two types of outcomes, whereas the \textbf{multinomial distribution} involves more than two categories. Suppose we have three types of mutually exclusive outcomes denoted by A, B, and C. Let \(P(A) = p_1, P(B) = p_2,\) and \(P(C) = p_3.\) In \(n\) independent trials, the probability of \(x_1\) outcomes of type \(A, \) \(x_2\) outcomes of type \(B,\) and \(x_3\) outcomes of type \(C\) is given by
\[
\frac{n!}{(x_1!)(x_2!)(x_3!)} \cdot p_1^{x_1} \cdot p_2^{x_2} \cdot p_3^{x_3}
\]
A genetics experiment involves six mutually exclusive genotypes identified as A, B, C, D, E, and F, and they are all equally likely. If 20 offspring are tested, find the probability of getting exactly five A’s, four B’s, three C’s, two D’s, three E’s, and three F’s by expanding the above expression so that it applies to six types of outcomes instead of only three.

40. The \textbf{Poisson distribution} applies to occurrences of some event over a specified interval, such as time or distance. The probability of the event occurring \(x\) times over an interval is given by
\[
P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!}, \text{ where } e \approx 2.71828
\]
and \(\mu\) is the mean number of occurrences over the interval. Over the past 100 years, the mean number of major earthquakes in the world for one year is 0.93. Assuming that the Poisson distribution is a suitable model, find the probability that the number of earthquakes in a randomly selected year is
\begin{enumerate}
  \item 0
  \item 1
  \item 2
  \item 3
  \item 4
  \item 5
  \item 6
  \item 7
\end{enumerate}